Chapter 1

Introduction

1.1 The van der Waals force

The history of the Casimir effect can be said to have begun around 1870, more than 35 years before Hendrik Casimir was born, when Johannes Diderik van der Waals wrote his doctor's thesis in which he derived a correction to the ideal gas law, now known as the van der Waals equation. The ideal gas law is

$$P = \frac{Nk_BT}{V}, \tag{1.1}$$

where P is the pressure, V is the available volume, N is the number of particles in the gas, k_B is Boltzmann's constant and T is the temperature. The van der Waals equation is

$$P = \frac{Nk_BT}{V - Nb} - \frac{n^2a}{V^2}, \tag{1.2}$$

where a represents an attractive interaction between the particles, and b is the volume of each particle. The physical rationale behind these corrections is that the volume available in the container is the volume of the container, minus the total volume occupied by the particles of the gas, and that the pressure is reduced by the interaction between the particles. This interaction is known as van der Waals forces, which is generally used as a collective term for all the forces between particles in a gas. The part that interests us, however, is the part known as the London force, or the London-van der Waals force, or simply as dispersion, which was the term Fritz London used.

The London force is the only force between atoms in a noble gas. Now, as we know, noble gases consist of neutral atoms with spherically symmetric charge distributions. What, then, is the origin of this force?

The charge distribution of a noble gas atom is on average spherically symmetric, but at any given moment there is a finite probability that the atom as a whole may have a dipole moment. If this is the case, this may lead to induced dipole moments in nearby atoms, which gives rise to a weak attraction.

In 1947, Casimir, together with Dirk Polder, wrote a paper titled "The influence of Retardation on the London-van der Waals Forces" [1], which was published in 1948. Retardation in this context refers to the fact that light, and hence the interaction between atoms, travels at finite speed. In this paper they calculated the force between a neutral atom and a perfectly conducting plane. Casimir later calculated the force between two neutral plates in a similar manner.

These calculations were rather complex, and went on for a large number of pages. Apparently, even Lifshitz commented that it was rather unwieldy [2]. The result, however, was a neat little expression,

$$\frac{\Delta E}{L^2} = -\hbar c \frac{\pi^2}{720} \frac{1}{d^3}. \tag{1.3}$$

1.2 Vacuum fluctuations

That the result of several pages of complex calculations should be such a simple expresson, devoid of all parameters but the distance between the plates and $\hbar$ and c puzzled Casimir, who mentioned this result to Niels Bohr. He later recalled that Bohr mumbled something about the vacuum [3], and so Casimir went and did the calculations again, but this time he looked at the vacuum fluctuations of QED.

To digress for a moment, vacuum fluctuations show up, for instance, when looking at the modes of the Klein-Gordon field as quantum harmonic oscillators, which is a commonly used introduction in textbooks. We will return to this in more detail later, but let us for now simply remember that such an oscillator never can have zero energy. This means that when we add up the ground state energies of all modes of the field, the result is an infinite energy density.

The concept that there exists an infinite amount of energy at all times everywhere may perhaps be philosophically unsatisfying, but does not present a problem for the predictive power of quantum field theory. As only differences in energy can be detected, the vacuum fluctuations have no effect on its ability to provide a useful description of nature. It has, however, led to a surprisingly large industry of conspiracy theorists, who claim that abundant free energy could be available to anyone, but that the technology is kept secret by Duracell.

The situation can, to attempt a metaphor, be compared to that of sitting in a small boat, out at sea: you are quite literally sitting on an ocean of potential energy, but as long as you do not have access to a lower potential, to which you can transfer the water, this energy is inaccessible. There is, however, one area of physics where this infinite energy can not be so easily dismissed, and that is general relativity. As energy produces curvature in space, an omnipresent infinite energy density should have dramatic and observable effects.

1.3 The cosmological-constant problem

Adding a constant energy density to the right hand side of the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{1.4}$$

is equivalent to adding a so-called "cosmological constant", $\lambda g_{\mu\nu}$, to the left hand side. The cosmological constant was originally added in an ad-hoc fashion by Albert Einstein, in order to allow his theory to describe a stationary universe, which was the reigning view at the time. A few years later, however, the universe was discovered by Edwin Hubble to be expanding, causing Einstein to discard his constant, famously calling it the biggest blunder of his life.

Recent measurements, however, seem to show that the universe is not only expanding, it is in fact expanding at an ever increasing rate. To account for this, the cosmological constant had to be reintroduced into Einstein's equation. As for the physical origins of this constant, the vacuum fluctuations would certainly seem to be a promising candidate. That is, until one does the math and discovers that the value predicted from quantum field theory is off by a ridiculous amount compared to the observed value.

This issue has been a source of much debate and quite a number of scientific publications, including a paper by Steven Weinberg, fittingly titled "The cosmological constant problem" [4]. I quote from his paper:

> As everyone knows, the trouble with this is that the energy density $\langle\rho\rangle$ of empty space is likely to be enormously larger than 10^{-47} GeV^4. For one thing, summing the zero-point energies of all normal modes of some field of mass m up to a wave number cutoff $\Lambda \gg m$ yields a vacuum energy density (with $\hbar = c = 1$)
>
> $$\langle\rho\rangle = \int_0^\Lambda \frac{4\pi k^2 \mathrm{d}k}{(2\pi)^3} \frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2} \tag{1.5}$$
>
> If we belive general relativity up to the Planck scale, then we might take $\Lambda \simeq (8\pi G)^{-1/2}$, which would give
>
> $$\langle\rho\rangle = 2^{-10}\pi^{-4}G^{-2} = 2 \times 10^{71}\mathrm{GeV}^4. \tag{1.6}$$
>
> But we saw that $|\langle\rho\rangle + \lambda/8\pi G|$ is less than about 10^{-47} GeV^4, so the two terms here must cancel to better than 118 decimal places.

1.4 The reality of the vacuum fluctuations

This is of course only an estimate, and the reality is more complex, but the general idea is that for things to add up, there must exist a huge cosmological constant, that almost, but not quite, cancels the energy density of the vacuum. Unlikely as this may seem, Weinberg never the less goes on to say

> Perhaps surprisingly, it was a long time before particle physicists began seriously to worry about this problem, despite the demonstration in the Casimir effect of the reality of zero-point energies.

Others, however, are more skeptical. Robert L. Jaffe, in a paper titled "Casimir effect and the quantum vacuum", writes:

> However, estimates of the energy density due to zero-point fluctuations exceed the measured value of λ by many orders of magnitude. Caution is appropriate when an

> effect, for which there is no direct experimental evidence, is the source of a huge discrepancy between theory and experiment.
> ...
> The object of this paper is to point out that the Casimir effect gives no more (or less) support for the reality of the vacuum energy of fluctuating quantum fields than any other one-loop effect in quantum electrodynamics, like the vacuum polarization contribution to the Lamb shift, for example. The Casimir force can be calculated without reference to vacuum fluctuations, and like all other observable effects in QED, it vanishes as the fine structure constant, α, goes to zero.

1.5 Back to Casimir

To return to Casimir, after his discussion with Bohr, he wrote a paper titled "On the attraction between two perfectly conducting plates" [5], where he made the first calculation on what is now known as the Casimir effect. This paper is a mere two and a half pages long, and does not contain any particularly difficult mathematics, yet the result was the same as before.

Because of its historical importance and its simplicity, we will go through the ideas of this paper in some detail. We begin by looking at a cube with sides L, bounded by perfectly conducting metal plates. Inside this cube, we place a square, perfectly conducting plate with sides L, parallel to the xy-plane and a small distance, d, away from it. The fact that the plate and the walls of the cube are perfectly conducting leads to boundary conditions. The electric field must vanish on the plates, and, hence, the only allowed frequencies inside the two cavities are those where an integer number of half wavelengths equals the distance between the walls:

$$k_x = \frac{\pi}{L} n_x, \;\; k_y = \frac{\pi}{L} n_y, \;\; k_z = \frac{\pi}{d} n_z, \tag{1.7}$$

where n_x, n_y, n_z, are positive integers. For each set of wavenumbers, there exists two different polarisations if n_x, n_y, $n_z > 0$, otherwise there is only one. However, for large L, we may regard k_x, k_y as continous variables to be integrated over, and hence the fact that there is only one polarisation state when $n_x =$ or $n_y = 0$ becomes negligible. After making the change to polar coordinates, we find that the sum of the zero-point energy in the cavity with volume $L \times L \times d$ is

$$\frac{1}{2} \sum \omega = \frac{L^2}{2\pi} \sum_{(0)\,1}^{\infty} \int r \mathrm{d}r \sqrt{n^2 \frac{\pi^2}{d^2} + r^2}, \tag{1.8}$$

where the notation $(0)\ 1$ indicates that for $n = 0$, we should multiply by $\frac{1}{2}$.

If we then increase d, so that it is on the order of $\frac{L}{2}$, we may also replace the sum over n by an integral, obtaining another infinite expression for the energy density. What interests us, however, is the difference between these two expressions:

$$\Delta E = \frac{L^2}{2\pi} \left\{ \sum_{(0)\,1}^{\infty} \int \sqrt{n^2 \frac{\pi^2}{d^2} + r^2} r \mathrm{d}r - \int \frac{d}{\pi} \sqrt{k^2 + x^2} r \mathrm{d}r \mathrm{d}k \right\} \tag{1.9}$$

To make sense of this result, we must apply some sort of trick. I quote from Casmir's paper:

> In order to obtain a finite result it is necessary to multiply the integrands by a function $f(k/k_m)$, which is unity for $k \gg k_m$, but tends to zero sufficiently rapidly for $(k/k_m) \to \infty$, where k_m may be defined by $f(1) = \frac{1}{2}$. The physical meaning is obvious: for very short waves (X-rays e.g.) our plate is hardly an obstacle at all and therefore the zero point energy of these waves will not be influenced by the position of this plate.

After a change of variables, and application of the Euler-Maclaurin summation formula, the result obtained is

$$\frac{\Delta E}{L^2} = -\hbar c \frac{\pi^2}{720} \frac{1}{d^3}, \tag{1.10}$$

a result that holds for $dk_m \gg 1$. I have included $\hbar$ and c, which are otherwise in this thesis set equal to unity, to emphasise the quantum nature of this effect.

The final result, as can be seen, is very simple. We might perhaps say that it is deceptively simple. Since the only parameters in the result are the fundametal constants $\hbar$ and c, and the separation between the plates, one might get the impression that the force is independent of any material-specific parameters. We must not forget, however, that this result is only valid for $dk_m \gg 1$, where k_m is highly material dependent.

1.6 The foretellings of Casimir

It is also interesting to look at a few closing comments Casimir made in this paper. In particular, it is interesting to look at the predictions he made, now that we have the advantage of hindsight.

> We are thus led to the following conclusions. There exists an attractive force between two metal plates which is independent of the material of the plates as long as the distance is so large that for wave lengths comparable with that distance the penetration depth is small compared with the distance. This force may be interpreted as a zero point pressure of electromagnetic waves.
>
> Although the effect is small, an experimental confirmation seems not unfeasable and might be of a certain interest.

In these two short paragraphs, Casmir touches upon several important points which have been the cause of much debate in recent years. Firstly, the aforementioned condition that must be satisfied for the force to be independent of the material. This is related to the obvious fact that there is no such thing as a perfect conductor at all frequencies, and hence no metal plate can stop all kinds of electromagnetic waves.

Secondly, he mentions that this force "may be interpreted as a zero point pressure of electromagnetic waves". And indeed, there is no lack of examples of those who, like Weinberg, claim that the Casimir effect proves the existence of the vacuum fluctuations present in quantum field theory. However, as Jaffe points out, the Casimir force can be calculated without any reference to the vacuum, and as we remember, this was indeed the way Casimir discovered this effect.

1.7 Measuring the Casimir effect

Regarding the feasibility of measurements of the Casimir effect, one might naively expect this to be a rather straightforward experiment. After all, all one would have to do is put up two parallel plates and measure the force between them. However, for the force to be measurable, the separation must be on the order of 1 μm, and the plates something on the order of 1 cm across. When considering that the force is expected to scale as $\frac{1}{d^4}$, it becomes obvious that while keeping the plates parallel is of the utmost importance if one is to obtain accurate results, it is also very difficult.

The first attempts to measure the Casimir effect, by Marcus Sparnaay [6] in the late fifties, used a simple setup of parallel plates. Steps were of course taken in order to ensure correct alignment of the plates, but, according to Sparnaay himself, even after perfoming the alignment procedure, there probably existed deviations from the parallel of about 0.2 μm. When measuring at an assumed separation of 0.5 μm this is quite significant, and while the results were not in disagreement with Casimir's calculation, neither were they very accurate.

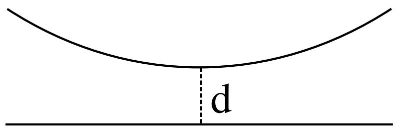

Figure 1.1: Casimir force setup of flat plate and part of a sphere. The only variable is d.

However, by means of the proximity force theorem [7, 8], one can obtain a simple expression,

$$F(d) = -\frac{\pi^3}{360} R \frac{\hbar c}{d^3} \tag{1.11}$$

for the force between a flat surface and a spherical one, provided the radius of curvature of the spherical surface, R, is large compared to the separation between the surfaces, d. When the curvature is known, such a system is completely described by only one variable, the separation at the closest point, as shown in Fig fig:plateogkule. Hence, one has eliminated any practical problems of keeping the plates parallel. Using this technique, Lamoreux[9], in 1997, and Mohideen and Roy[7], in 1998, were able to measure the Casimir effect quite accurately.

Lamoreaux used a setup with a torsion pendulum, suspended from a thin tungsten fiber. A flat plate was mounted to one side of the pivot point, and this was brought close to a spherical surface with a large radius of curvature, on the order of 10 cm. To the other side of the pivot, two capacitor plates were placed, one on each side of the pendulum. The voltage across the plates can be varied, and when this voltage is adjusted exactly to compensate for the Casimir force, the pendulum will be at rest, and the Casimir force can be calculated.

Measurements were performed down to a separation of about 0.6 μm, and very good agreement with the theory was found. Lamoreaux estimates the accuracy to be on the order of 5%, and states that "The vacuum stress between closely spaced conducting surfaces, due to the modification of the zero- point fluctuations of the electromagnetic field, has been conclusively demonstrated.".

Mohideen and Roy, following a different approach, used a polystyrene sphere with a radius of 100 μm, and plated with a layer of aluminium and a thin layer of gold and palladium to prevent oxidation (see Fig. 1.2). The sphere was mounted on the cantilever of an Atomic Force Microscope (AFM), and the force could be readily calculated with Hooke's law when the deflection of the cantilever was known. This was found by shining a laser on the back of the cantilever and measuring the position of the reflected beam, which is the normal way of using an AFM.

By this method, Mohideen and Roy were able to measure the Casimir force between the sphere and a similarly plated piece of polished sapphire for separations down to only 120 nm. They estimate the error in their results to be on the order of 1%, and excellent agreement with theory, including corrections for finite temperature, roughness and conductivity, was found.

1.8 The current state of affairs

The last point mentioned by Casimir is that that the result of such measurements may be of some interest, and this has indeed turned out to be the case. Casimir physics is today an active field of research with a rather large number of publications per year. The research being done ranges from fundamental theoretical considerations, to the investigation of several possible applications in nano technology.

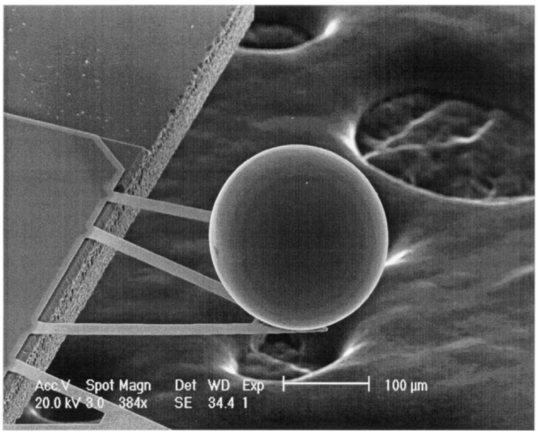

Figure 1.2: Metal plated polystyrene sphere of radius 100 μm, mounted on AFM-cantilever. (Image taken from [7], and used with permission from U. Mohideen.)

The relevance to nano technology should not be unexpected: the Casimir effect will be the dominant force between neutral metallic surfaces at small separations and it is therefore obvious that it is important to have a thorough knowledge of its properties if one wants to design machines with moving parts on a very small scale. Unless such things are taken into consideration, the Casimir force might quite literally make the parts of one's machine stick together.

An example of the "sticking power" of the Casimir effect may already be found in nature. The gecko, with its ability to walk, and even run, hanging upside down from smooth surfaces like glass, has long been a puzzle. The gecko uses neither suction, nor any gripping mechanism, and it does not leave behind any residue from a sticky substance. So how does it manage this feat?

It has been known for some time that the gecko's feet are covered in extremely small, leaf-shaped hairs, which get very close to the surface it walks on. The leading theories have been that either capillary action or van der Waals' forces is responsible, and in 2002, Kellar Autumn *et. al.* [10] showed that the force is the same on hydrophobic and hydrophilic surfaces, meaning it has to be van der Waals' forces, which is closely related to the Casimir force.

However, aside from hindering the operation of small machines, and in addition to providing the idea for a new kind of adhesive tape, the Casimir force might also have other useful applications. The force is highly dependent upon the reflective properties of the surfaces involved, which to a large degree can be made to desired specifications by the use of various coatings.

Among the more exotic examples, research is being done on the Casimir force on hydrogen switchable mirrors, which are polished metal surfaces that become transparent when absorbing hydrogen from the atmosphere [11, 12].

Research is also being done on the effects of phase transitions in superconductors on the Casimir force, and on the forces between materials with exotic optical properties. In particular, recent results by Ulf Leonhardt and Thomas Philbin[13], indicate that sandwiching a perfect lens between two metal plates might lead to repulsive Casimir forces. This last example also sparked wider interest, with articles appearing in regular newspapers like the Daily Telegraph, even though the article was accompanied by a completely unrelated picture of a magnetically levitating spinning top and jumped to conclusions about levitating humans. Articles treating the Casimir effect in a more general way have also begun appearing in non-specialist publications, like for instance The Economist[14].

That the Casimir effect may be entering into the vocabulary of laypeople is certainly an indicator of the interest this field has generated among scientists, and considering the amount of attention that goes to nano technology these days, the future seems bright for a field emerging from an article that was mostly seen as a theoretical curiosity when it was published, 60 years ago this year.

Bibliography

[1] H. Casimir and D. Polder. *Physical Review* **73**(4) (1948).

[2] S. Lamoreaux. *Physics Today* **2** (2007).

[3] H. Casimir. *Comments on modern physics* **2**(5-6) (2000).

[4] S. Weinberg. *Rev. Mod. Phys.* **61**(1) (1989).

[5] H. B. G. Casimir. *Proc. K. Ned. Akad. Wet.* **51**, 793 (1948).

[6] M. Sparnaay. *Physica* **24**(6-10) (1958).

[7] U. Mohideen and A. Roy. *Phys. Rev. Lett.* **81**(21), 4549–4552 (1998).

[8] C. F. Tsang, W. J. Swiatecki, J. Randrup, and J. Blocki. *Annals of Physics* **105** (1977).

[9] S. K. Lamoreaux. *Phys. Rev. Lett.* **78**(1), 5–8 (1997).

[10] K. Autumn *et al.* *PNAS* **99**(19) (2002).

[11] D. Iannuzzi, M. Lisanti, and F. Capasso. *PNAS* **101**(12) (2004).

[12] S.de Man and D. Iannuzzi. *New Journal of Physics* **8**(235) (2006).

[13] U. Leonhardt and T. G. Philbin. *New Journal of Physics* **9** (2007).

[14] *The Economist* **May 22nd** (2008).